Proportional Return of Interventions: Cost-Effective Surveillance for the Early Detection of Gypsy Moth

Technical Report for CEBRA project 19NZ03

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Centre of Excellence for Biosecurity Risk Analysis

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1. Introduction

Protecting New Zealand's natural environment and agricultural industries from invasive species has long been a priority of the Ministry for Primary Industries (MPI). It is well known that prevention and early detection is the best approach for many pests due to the high costs that eradication programs carry and the significant damages that come with an incursion.

This project covers a fourth year of work, building on the work completed in previous years for projects 170621[1] and 1606E[2] . The first and second years of the project established a framework for representing the biosecurity system across three main areas (pre-border, border and post-border) with four main pathways (craft, cargo, mail and passenger) overlaid with the seven groups of biosecurity risk assessment/management activities as identified in Schneider *et al.* (2020) - (Anticipate, Prevent, Screen, Prepare, Detect, Respond and Recover). The third year of the project tested and finalised the biological component (i.e. estimates of risk reduction across intervention activities of the system) of the risk decision support tool by running a selected set of five priority pests identified by MPI through the matrix. This final stage of the project looks at overlaying an economic cost of interventions to provide an overview of the greatest return (risk reduction) on investment. We consider the post border activities in particular and propose a model for assessing the optimal surveillance of a pest using traps. We apply this to a case study of Gypsy Moth but it may be applied to a number of other pests on MPI's priority list[3] such as fruit flies (e.g. Queensland Fruit Fly, Oriental Fruit Fly, Medfly), ants (e.g. Red Imported Fire Ant) and other moths (e.g. Nun Moth, Painted Apple Moth) for which traps are also used for surveillance.

A key benefit of this model is that it requires a small number of parameters so is very useful in a practical setting where data is limited. It can help to determine a surveillance strategy without the need for complex habitat suitability models and other information that is not readily available in practice. Our model extends the work outlined in Kompas *et al.* (2016) in which they developed a simple model for determining optimal surveillance of sleeper weeds. This project adapts the model so that it is suitable for pests that are detected using traps.

This report begins in Chapter 2 by introducing the economic model that we will be using in a general sense. We start by showing the cost of an incursion of a known size and we then go on to show our detection probability function and finally tie it all together by defining the cost of multiple incursions and the optimisation problem that we use to determine optimal surveillance. In Chapter 3, we apply the model to Gypsy Moth and discuss the results. Finally, Chapter 4 acts as a space for discussion and conclusions.

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[1] https://cebra.unimelb.edu.au/__data/assets/pdf_file/0007/3350374/CEBRA_170621_Yr2_Final.pdf

[2] https://cebra.unimelb.edu.au/__data/assets/pdf_file/0012/2946558/Final-Report-1606E.pdf

[3] https://www.mpi.govt.nz/protection-and-response/finding-and-reporting-pests-and-diseases/priority-pests-plant-aquatic/

2. Economic Model

This chapter describes an economic model for Gypsy Moth incursions which are discovered using a static surveillance grid. The method outlined in this chapter closely follows the method used by Kompas *et al.* (2016) with the complex mathematics presented in Appendix A for the more technical reader. Appendix A follows the same narrative as this chapter so if the reader desires more mathematical context on a particular aspect of the model, it is clear which section each equation relates to. There are certain essential parameters that this model relies on as well as key variables that we will be using throughout this chapter. We define these as follows:

Table 2.1.: Parameter and variable definitions

Parameter /Variable	Type	Definition
x_0	Parameter	Infestation size of an initial incursion
r	Parameter	Infestation growth rate
ρ	Parameter	Economic discount rate
c	Parameter	Cost of eradication per unit area
d	Parameter	Damage per unit area per unit time
l	Parameter	Detection radius from trap
b	Parameter	Mean interval between incursions
σ	Parameter	Standard deviation of interval between incursions
T	Variable	Time
$x(T)$	Variable	Size of the infestation at time T
s	Variable	Surveillance expenditure
$y(s)$	Variable	Grid size for surveillance expenditure s

All parameters and variables must use the same scale in both area and time. We first calculate the cost of a known incursion size, then outline how the detection probability would be calculated and finally combine these two sections to calculate the total cost of multiple incursions.

2.1. Cost of an incursion of known size

We consider a particular land parcel that may have traps on it or may not. An infestation of size x_0 establishes within the land parcel area with some frequency b. The size of the infestation has growth rate $r > 0$ and so we can calculate the size of the infestation at time T as a function of x_0, r and T which we call $x(T)$:

$$x(T) = x_0 e^{rT} \tag{2.1}$$

Throughout this section, we calculate everything at a given time T. Once we know the size of the infestation, we can calculate the eradication cost at time T as a function of ρ, c, T and $x(T)$ where c is the cost of eradication per unit area and ρ is the discount rate. This we call $R(T)$:

$$R(T) = \underbrace{\text{eradication cost per unit area } \times \text{ size of infestation when it is discovered}}_{\text{discounted to today's money}} \tag{2.2}$$

The second cost that needs considering is the losses associated with an infestation. These may be avoided to some extent by choosing a strategy of early eradication and are cumulative; so, the more time that the Gypsy Moth is present, the more losses are incurred. These may be environmental, agricultural, social or one of many other factors that might be affected. We parametrise all these damages and say that d is the damage per unit area per unit time. Since damages are cumulative, we must consider all damages from time 0 when the incursion first started to time T when eradication is successful. We define the present value of the losses associated with an incursion as:

$$L(T) = \underbrace{\text{damages per unit area per unit time} \times \text{size of infestation}}_{\text{across all times that the infestation is present, discounted to today's money}} \tag{2.3}$$

Kompas *et al.* (2016) showed how the condition for which immediate eradication is efficient is

$$d + cr > c\rho \tag{2.4}$$

and the same is true in this case. In particular, if the damages are sufficiently large and the cost of eradication would increase more than your money could increase by investing then it is best to eradicate as soon as the pest is discovered. This equation is derived from the first order conditions for cost minimisation by optimising over the time to eradication.

2.2. Detection probability

The cost of controlling a known invasion is $R(T) + L(T)$. Both of these aspects depend on the size of the incursion when it is discovered which is a direct result of surveillance effort. Surveillance is not 100% effective and we use a detection probability function to represent this.

We use a practical detection function that should be more useful in practice than many standard probability distributions since there is no need to estimate abstract parameters that have little grounding in real life.

In practice, high priority areas where surveillance activity takes place have pheromone traps placed on preferred host trees in a grid pattern. In fact, the area is divided into cells of a certain size and the traps are placed on a tree at the centre of each grid cell. The traps can effectively lure Gypsy Moths from a certain distance with some probability so when the traps are examined, MPI will be notified of a Gypsy Moth in the area and further investigation can take place and an eradication programme can commence.

A number of studies have been done into the effectiveness of Gypsy Moth traps. Schwalbe (1981) and Keyes (1997) tested the proportion of moths returned for a given intertrap distance (ITD). For our purpose, in order for a trap to effectively alert us to an infestation, we need just one moth to be caught. Gypsy Moths lay approximately 500-1000 eggs at a time (Glare *et al.*, 2003) so if an egg mass is at a distance l from a trap whereby the probability that a moth will be caught at distance l is 0.01, then at least one moth will be caught with probability 0.99. We say that l is the minimum distance from the trap where an infestation would be successfully detected and call it the detection radius. As such, the trap detection is as depicted in Figure 2.1. In New Zealand, areas are identified as being good for trapping and the land is divided into a grid. Traps are then placed on trees as centrally to the grid cell as possible, as illustrated in Figure 2.1. In fact, if we assume that the incursion takes the shape of a circle with radius k, then the probability that the incursion is detected is the proportion of the grid that is covered by the circle with radius $l + k$.

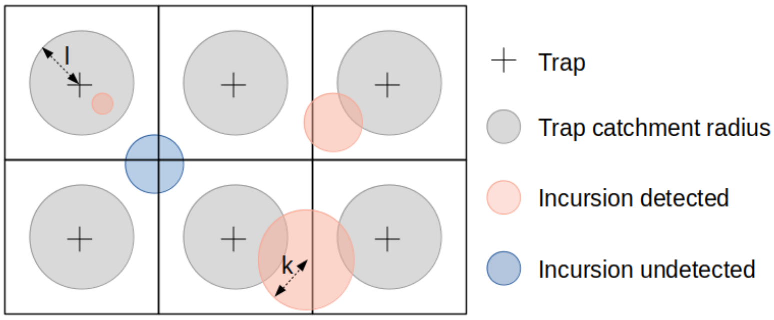


Figure 2.1.: Demonstration of trap detection effectiveness.

To formulate this mathematically, we consider the three scenarios outlined in Figure 2.2. In particular, although the detection radius l is fixed, as we vary the surveillance budget, the length of the grid cells y may increase or decrease. In Figure 2.2, the grey areas indicate the portions of the grid cell where an incursion would not be detected. Calculating the proportion of the square that is covered by the circle is trivial for scenarios (a) and (c) but it gets more complicated in scenario (b) when we need to account for the segments of the circle that are outside the cell area. In practice, there are many grid cells that are adjacent to other grid cells and therefore the probability of detection would increase in scenario (b) where the trapping areas overlap. For modelling purposes, we treat each grid cell as independent. For full details on how the detection function is calculated for scenario (b), see Appendix B.

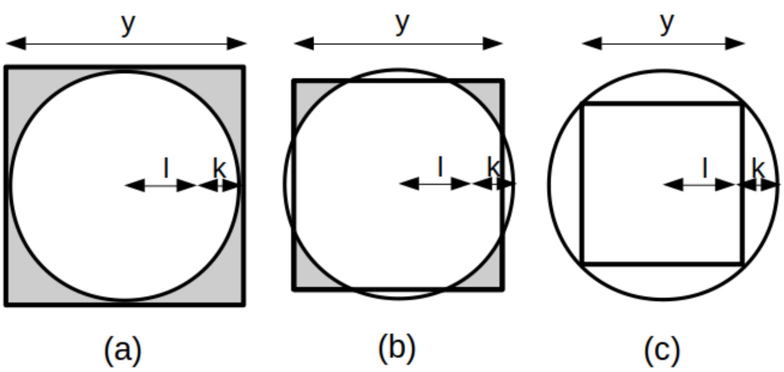
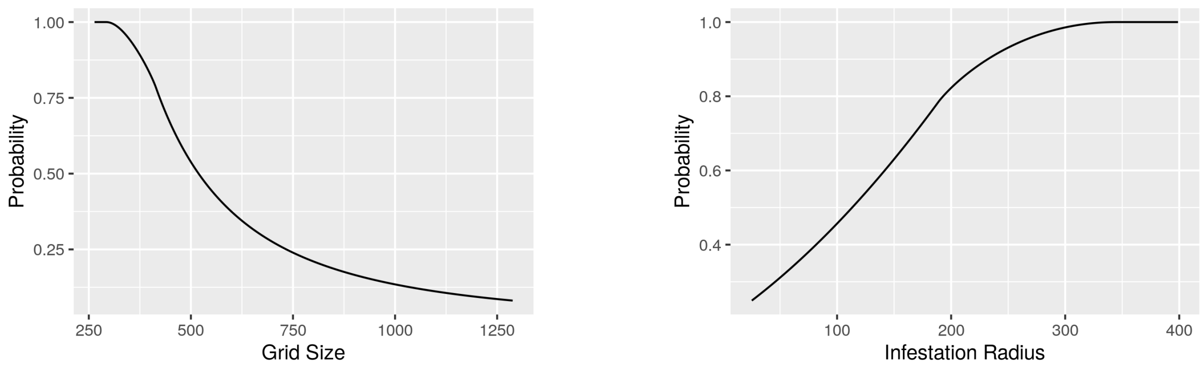


Figure 2.2.: Three scenarios for calculating detection probability.

In fact, whichever scenario we are considering, we can define the probability of detection as a function of $x(T)$, y and l:

$$p(x, y) = \frac{\text{area of the circle where detection radius intersects with incursion}}{\text{area of grid cell}} - \text{area of segments outside grid cell} \tag{2.5}$$

Figure 2.3 plots the marginal probabilities from Equation A.6. Figure 2.3a fixes the infestation radius $k = 21$m and varies the grid size y along the horizontal axis. Similarly, Figure 2.3b fixes the grid size $y = 750$m and varies the infestation radius k along the horizontal axis. In both plots, the detection probability $p(x, y)$ is shown on the vertical axis and, as would be expected, the functions are monotonic and continuous and take values only in the interval $[0, 1]$.



(a) Detection probability with fixed $k = 21$m

(b) Detection probability with fixed $y = 750$m

Figure 2.3.: Plots of the marginal probabilities from Equation A.6.

2.3. Total cost of multiple incursions

The previous equations have all been for a single incursion; we now consider the subsequent problem whereby there are a series of entries arriving approximately at a given interval b. We use the method proposed in Kompas *et al.* (2016) where the entry times of the species are denoted as a random walk process. Then, for a given surveillance expenditure s, we combine $L(T)$, $R(T)$ and $p(x, y)$ for all entries to get the present value of all the expected damages and eradication expenditure across all incursions:

$$C(s) = \underbrace{\text{expected eradication costs + expected damages}}_{\substack{\text{given when we think we would find the incursion, for all} \\ \text{time, discounted to today's money, for all incursions, given} \\ \text{a set surveillance budget}}} \tag{2.6}$$

We present the cost as a function of surveillance expenditure rather than, say, grid size because it is a metric that decision makers are familiar with at all levels and it enables people to clearly see the connection between investment in surveillance and expected eradication costs and damages. The final cost that must be considered is that of the surveillance programme itself. This is more simple to calculate and may be parametrised as constant s. Now that we have all the individual pieces, we can say that the optimal surveillance budget is that which minimises the following:

$$\underset{s \geq 0}{\text{minimise}} \; (s + \rho C(s)) \tag{2.7}$$

The benefit of considering the problem in terms of unique land parcels is that it allows us to easily account for the fact that different geographical locations will hold different likelihoods and consequences of an incursion and this information can be captured in the input parameters. An additional convenience of this model is that it is unit-free and the land parcels can be to any scale that we choose as long as the units are consistent across all the parameters.

3. Gypsy Moth

We applied the model outlined in Chapter 2 to a Gypsy Moth case study. Gypsy Moth is a problematic pest in many countries around the world (McEwan *et al.*, 2009; Liebhold *et al.*, 1992; Elkinton & Liebhold, 1990) and is currently not established in New Zealand. It is believed that were Gypsy Moth to be introduced to New Zealand, it would have a devastating effect on the natural environment and agricultural industries (Matsuki *et al.*, 2001; Pitt *et al.*, 2007; Walsh *et al.*, 1993; Glare *et al.*, 2003). New Zealand currently uses traps for early detection of Gypsy Moth (Ross, 2005; Kean *et al.*, 2008) which so far have been successful at keeping the pest out. Gypsy Moth traps use pheromones (disparlure) to attract the male Gypsy Moths (Sharov *et al.*, 2002) and the traps are inspected fortnightly. The trap placement in New Zealand has particular focus on airports, ports and large population centres.

3.1. Parameters

It is common in the field of biosecurity that rich data that would be needed to train complex models is not available (Burgman *et al.*, 2011), so it is standard practice in such a scenario to use expert judgement as a source of data (Sutherland, 2006; Sutherland & Burgman, 2015). The previous phase of this project carried out substantial expert elicitation activities in the area of Gypsy Moth so we were able to use these outputs directly. The experts gave an estimate of the size of an initial incursion under the current surveillance so we determined that $x_0 = 1344.60$ hectares.

They carried out some analysis on the experts' estimations of approach rate and determined that there is an approximate rate of an establishment every 61 years; therefore, $b = 61$. The experts also gave estimations of damages for a given incursion size and eradication costs for a given incursion size, thus we inferred that $c = 0.65$ ($ per square metre) and $d = 0.29$ ($ per square metre per year).

We used a spread rate of $r = 0.26$ (Leuschner *et al.*, 1996) and a discount rate of $\rho = 0.03$ as is standard in environmental problems. We note that with these parameters, the conditions for Equation A.5 do indeed hold so it is preferable to eradicate Gypsy Moth as soon as it is discovered.

We used MPI data that recorded the current surveillance programme costs as being $408,860 per year, $260,678 of which are variable costs. Fixed costs cover the general cost of having a surveillance programme at all and these costs are not insignificant. Variable costs are items such as the cost of the traps themselves and the salaries of the people who inspect the traps - these will vary depending on the scale of the surveillance programme. The data also showed that 1,525 individual traps are part of this surveillance programme and that each grid cell measures 750 x 750 metres. Using this information, we calculated a cost per trap of $171 and a total trapping area of 858 square kilometers. We kept the total trapping area constant when varying the surveillance effort so that an increased surveillance budget meant finer grid cells rather than more area covered by traps. We did this for a number of reasons, not least because land parcels cannot be assumed to be the same in terms of risk; we must assume that the traps that are currently in use are in the most susceptible areas and therefore the most useful for trapping.

Finally, we needed information on the trap's effectiveness at luring an individual moth. We used data from release-recapture experiments carried out (Schwalbe, 1981; Keyes, 1997) to train a log-linear model. We then used this model to predict the radius which recaptures 1% of Gypsy Moths and found that the detection radius was around $l = 186$m.

3.2. Optimal surveillance

We ran the model from Chapter 2 with the parameters from Section 3.1 and obtained a solution as demonstrated in Figure 3.1. Here, we see a somewhat flat version of the familiar U-shaped function where damages and eradication costs decrease before rising surveillance costs drag us back out of optimality. We see that the optimal budget is around $603,000.

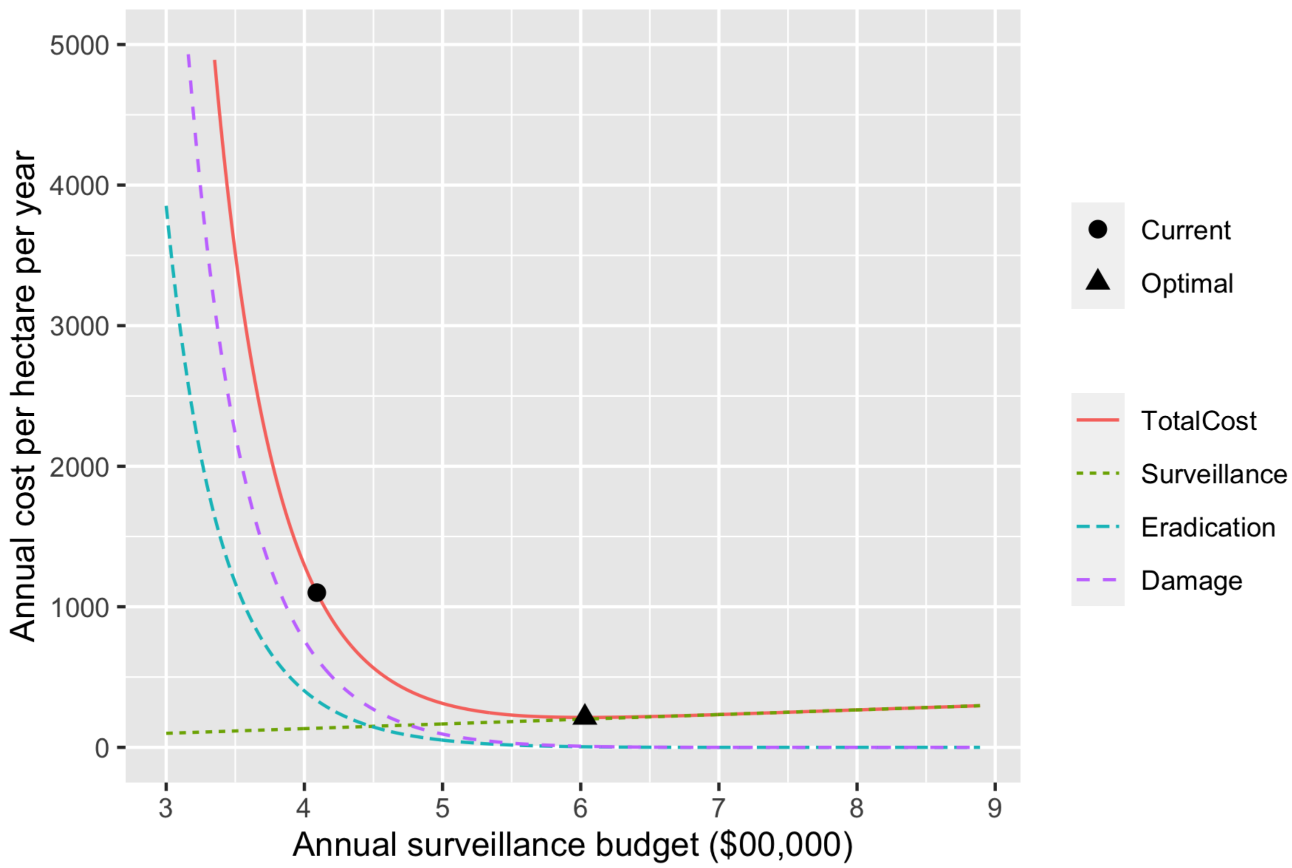


Figure 3.1.: Surveillance trade-off between cost components for Gypsy Moth

We carried out sensitivity analysis on some of the parameters to determine how stable our solution was and how precise our parameter estimates would need to be. Firstly, we compared the sensitivity of entry rate and entry size. Table 3.1 shows the results of this analysis and we see that the results do not vary substantially when these parameters change.

Table 3.1.: Sensitivity analysis for entry rate (b) and entry size (x_0)

	Entry rate (b)		
Entry size (x_0)	50	61	72
800	603000	601000	600000
1344.6	605000	603000	602000
1800	606000	604000	603000

Next, we did sensitivity analysis on the spread rate and discount rate. We see a slightly larger deviation from the original solution here but not significantly so. The results can be seen in Table 3.2. Interestingly, an increase in spread rate does not equate to an increase in optimal surveillance budget. This is due to the large initial incursion size so as the spread rate increases, the incursion will be found with certainty more quickly, without the need for additional traps.

Table 3.2.: Sensitivity analysis for spread rate (r) and discount rate (ρ)

	Spread rate (r)		
Discount rate (ρ)	0.2	0.26	0.32
0.02	600000	595000	591000
0.03	607000	603000	600000
0.04	613000	610000	607000

We then varied the damage and eradication costs per unit area. We found that again there was a small deviation but nothing significant. The results are outlined in Table 3.3

Table 3.3.: Sensitivity analysis for damage (d) and eradication (c) costs per unit area

	Damage cost per unit area (d)		
Eradication cost per unit area (c)	0.23	0.29	0.35
0.55	595000	601000	607000
0.65	597000	603000	609000
0.75	600000	606000	611000

Finally, we varied the detection radius and the cost per trap. In Table 3.4, we see far more sensitivity than in the other variables. Our estimation of detection radius was based on a small dataset, however most of the points cluster around an ITD of 125-375 (detection radius 62.5-187.5) so our estimate of detection radius should be reasonably accurate for the 0.01 threshold. However, this threshold was chosen as the lower bound on the size of an egg mass which can be between 100-1200 eggs. If we based our parameter choices on an egg mass of 1200 eggs, the threshold would be much lower and therefore the detection radius would be much larger. However, without more data on the population density of Gypsy Moths, this is the best we will be able to do.

Additionally, the current surveillance budget of $408860 does not sit in the range of even the most sensitive estimates, indicating that directing additional funding towards

Table 3.4.: Sensitivity analysis for detection radius and cost per trap

	Detection radius		
Cost per trap	170	186.5	200
160	602000	577000	558000
170	627000	601000	582000
180	652000	625000	605000

Gypsy Moth surveillance through smaller grid cell sizes would save valuable time and is cost-effective in the event of an incursion.

4. Discussion & Conclusions

This project applied an economic lens to part of an existing risk model and considered optimal surveillance from a rate of return perspective rather than purely risk based. We developed a new, highly practical detection probability function for pests that are detected using traps and applied the whole model to Gypsy Moth. The model showed us that putting more funding towards Gypsy Moth surveillance would yield a greater rate of return. In particular, carrying out surveillance in a finer grid would improve detectability in those areas.

Further work might include applying a similar rate of return approach to other areas of the model developed in the earlier stages of the project such as pre-border and border. It would also be interesting to consider eradication feasibility and the cut-off when a Gypsy Moth infestation becomes too large for it still to be economically efficient to eradicate an infestation. An extension of this work could also include a Monte Carlo simulation of the biophysical parameters (time to detection) under the current grid and the optimal grid. Additionally, testing this method on other species of interest such as Queensland Fruit Fly or Red Imported Fire Ant would be interesting; since the model requires so few parameters, it is widely adaptable to many applications. Finally, further investigation could be done into the trade off between increasing grid fineness and commencing surveillance in a new area entirely. Given sufficient data about the characteristics of different land parcels, a variation of this tool could be used to determine optimal surveillance across multiple sites.

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A. Economic Model

The mathematics in this appendix borrows significantly from Kompas *et al.* (2016) with some additional explanation and modest adaptation.

A.1. Cost of an incursion of known size

We consider a particular land parcel that may have traps on it or may not. An infestation of size x_0 establishes within the land parcel area with some frequency b. The size of the infestation has growth rate $r > 0$ so we might say that at time T, the infestation is of size

$$x(T) = x_0 e^{rT} \tag{A.1}$$

The cost of eradication varies with the size of the infestation. If c is the cost of eradication per unit area, then the discounted total eradication cost at time T with discount rate ρ is

$$R(T) = e^{-\rho T} cx(T) \tag{A.2}$$

Here, $cx(T)$ is the actual cost of eradication at the time it is taking place. In order to compare costs across different times, we discount everything to what it would be worth in today's money so that we can make a fair comparison. The $e^{-\rho T}$ term represents what the money could have been worth were it invested instead of being spent on, in this case, the eradication of Gypsy Moth from an area of New Zealand. We substitute $x(T)$ using Equation A.1 so that the simplified, discounted cost of eradication is

$$R(T) = cx_0 e^{(r-\rho)T} \tag{A.3}$$

The second cost that needs considering is the losses associated with an infestation. These may be avoided to some extent by choosing a strategy of early eradication and are cumulative; so, the more time that the Gypsy Moth is present, the more losses are incurred. These may be environmental, agricultural, social or one of many other factors that might be affected. We parametrise all these damages and say that d is the damage per unit area per unit time. Since damages are cumulative, we must consider all damages from time 0 when the incursion first started to time T when eradication is successful. As such, we integrate over this time period and obtain the present value of the loss associated with an incursion which is defined as

$$L(T) = \int_0^T [dx(t)]e^{-\rho t} dt = x_0 \frac{d}{r-\rho}[e^{(r-\rho)T} - 1] \tag{A.4}$$

Note that the $e^{-\rho t}$ term is being used again to discount the costs to the present value and the $dx(t)$ term is the future value of the damages incurred at time t.

Kompas *et al.* (2016) showed how the condition for which immediate eradication is efficient is

$$d + cr > c\rho \tag{A.5}$$

and the same is true in this case. In particular, if the damages are sufficiently large and the cost of eradication would increase more than your money could increase by investing then it is best to eradicate as soon as the pest is discovered.

A.2. Detection probability

The cost of controlling a known invasion is the sum of Equation A.3 and Equation A.4. Both of these aspects depend on the size of the incursion when it is discovered which is a direct result of surveillance effort. Surveillance is not 100% effective and we use a detection probability function to represent this.

We use a practical detection function that should be more useful in practice than many standard probability distributions since there is no need to estimate abstract parameters that have little grounding in real life.

In practice, high priority areas where surveillance activity takes place have pheromone traps placed on preferred host trees in a grid pattern. In fact, the area is divided into cells of a certain size and the traps are placed on a tree at the centre of each grid cell. The traps can effectively lure Gypsy Moths from a certain distance with some probability so when the traps are examined, MPI will be notified of a Gypsy Moth in the area and further investigation can take place and an eradication programme can commence.

A number of studies have been done into the effectiveness of Gypsy Moth traps. Schwalbe (1981) and Keyes (1997) tested the proportion of moths returned for a given intertrap distance (ITD). For our purpose, in order for a trap to effectively alert us to an infestation, we need just one moth to be caught. Gypsy Moths lay approximately 100-1200 eggs at a time so if an egg mass is at a distance l from a trap whereby the probability that a moth will be caught at distance l is 0.01, then at least one moth will be caught with probability 1. We say that l is the minimum distance from the trap where an infestation would be successfully detected and call it the detection radius. As such, the trap detection is as depicted in Figure A.1. In fact, if we assume that the incursion takes the shape of a circle with radius k, then the probability that the incursion is detected is the proportion of the grid that is covered by the circle with radius $l + k$.

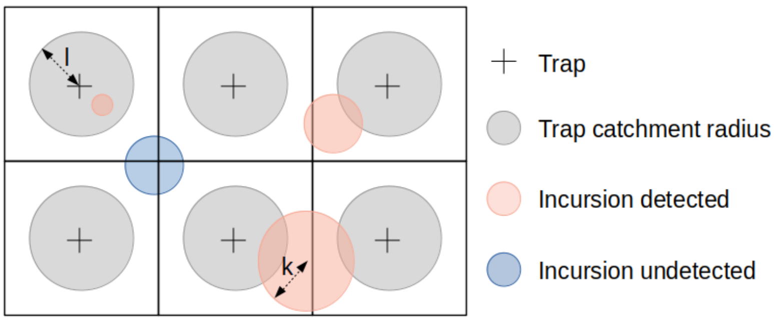


Figure A.1.: Demonstration of trap detection effectiveness.

To formulate this mathematically, we consider the three scenarios outlined in Figure A.2. In particular, although the detection radius l is fixed, as we vary the surveillance budget, the length of the grid cells y may increase or decrease. In Figure A.2, the grey areas indicate the portions of the grid cell where an incursion would not be detected. Calculating the proportion of the square that is covered by the circle is trivial for scenarios (a) and (c) but it gets more complicated in scenario (b) when we need to account for the segments of the circle that are outside the cell area. For full details on how the detection function is calculated for scenario (b), see Appendix B.

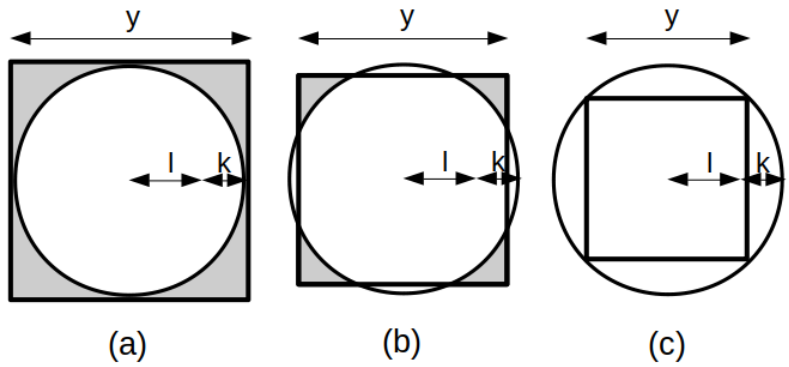


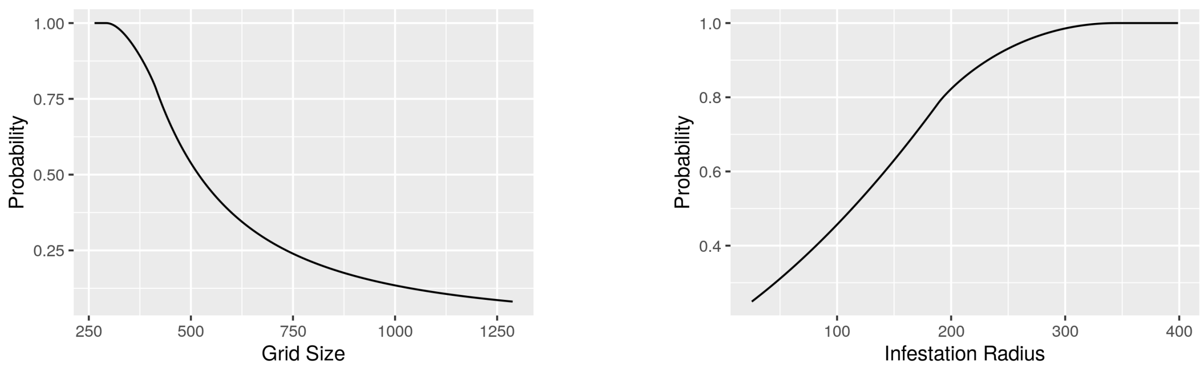
Figure A.2.: Three scenarios for calculating detection probability.

In practical terms, (a) shows the scenario with the largest grid size where the traps are furthest apart from each other; in contrast, (c) shows the scenario where the grid size is sufficiently small that there is some overlap between adjacent traps. We use these three scenarios to calculate the detection probability function as in Equation A.6. The first scenario, where $l + k \leq \frac{y}{2}$, relates to scenario (a) in Figure A.2. Likewise, the second scenario where $\frac{y}{2} < l + k < \frac{\sqrt{2y^2}}{2}$ relates to scenario (b) and the third scenario with $l + k \geq \frac{\sqrt{2y^2}}{2}$ relates to scenario (c).

In Equation A.6, we assume that l is fixed, and calculate the probability of detection for various cell lengths (y) and infestation sizes (x) with $x = \pi k^2$.

$$p(x,y) = \begin{cases} \frac{\pi(l+k)^2}{y^2}, & \text{if } l+k \leq \frac{y}{2} \\ \frac{\pi(l+k)^2 - 4\cos^{-1}\left(\frac{y}{2(l+k)}\right)(l+k)^2 + 2y\sqrt{(l+k)^2 - \left(\frac{y}{2}\right)^2}}{y^2}, & \text{if } \frac{y}{2} < l+k < \frac{\sqrt{2y^2}}{2} \\ 1, & \text{if } l+k \geq \frac{\sqrt{2y^2}}{2} \end{cases} \tag{A.6}$$

Figure A.3 plots the marginal probabilities from Equation A.6. Figure A.3a fixes the infestation radius $k = 21$m and varies the grid size y along the horizontal axis. Similarly, Figure A.3b fixes the grid size $y = 750$m and varies the infestation radius k along the horizontal axis. In both plots, the detection probability $p(x,y)$ is shown on the vertical axis and, as would be expected, the functions are monotonic and continuous and take values only in the interval $[0,1]$.



(a) Detection probability with fixed $k = 21$m

(b) Detection probability with fixed $y = 750$m

Figure A.3.: Plots of the marginal probabilities from Equation A.6.

A.3. Total cost of multiple incursions

The previous equations have all been for a single incursion; we now consider the subsequent problem whereby there are a series of entries arriving approximately at a given interval b. We use the method proposed in Kompas *et al.* (2016) where the entry times of the species are denoted as a random walk process:

$$t_i = t_{i-1} + b + \epsilon_i \quad \text{for } i = 1, \ldots, \infty; \quad t_0 \equiv 0 \quad \text{and } \epsilon \sim \mathcal{N}(0, \sigma^2) \tag{A.7}$$

Here, t_i is the time of the ith entry, b is the approximate interval between entries and ϵ is an independent, normally distributed random variable that allows for variation in the arrival interval. Then we combine Equations A.3, A.4 and A.6 for all entries to get the present value of the expected damages and eradication expenditure:

$$\begin{aligned} C(s) &= \underbrace{\sum_{i=1}^{\infty}}_{\text{for all incursions, for all time}} E_{t_i} \underbrace{\{e^{-\rho t_i}}_{\text{discounted to today's money}} E_{x_i} \{ \underbrace{L(T(x)) + R(T(x))}_{\text{eradication + damages given when we think we would find the incursion}} \underbrace{|(t_i, y(s))}_{\text{given a set surveillance budget}} \} \} \\ &= \sum_{i=1}^{\infty} E_{t_i} \{e^{-\rho t_i} \int_{x_0}^{\infty} [L(T(x)) + R(T(x))] \frac{\partial p(x, y(s))}{\partial x} dx\} \end{aligned} \tag{A.8}$$

Here, E_{t_i} and E_{x_i} are the expectation operators for t_i and x_i respectively and $T(x)$ is the inverse of Equation A.1. The first row of Equation A.8 is a summation over time of the expected value of damages ($L(T(x))$) and eradication costs ($R(T(x))$). Within the first expectation operator E_{t_i}, the expectation with respect to x_i is discounted to the present value as in previous equations. The key difference in this discounting to what we have seen previously, however, is that now it is discounting to the present to allow for multiple incursions over a long time; whereas, previously, it was just discounting to the start of the incursion. The second row of Equation A.8 summarises E_{x_i} in the form of an integral. In particular, the expected value of a continuous random variable X is calculated as:

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx \tag{A.9}$$

where $f(x)$ is the probability density function (PDF) of X. In our case, Equation A.6 gives us the probability values for varying x and y; in order to obtain the PDF from this, we partially differentiate with respect to x and hence obtain the result in the second row of Equation A.8. We integrate across all times that the incursion may be discovered - in particular, it may be discovered as soon as it arrives or it may continue to grow until the point of natural detection.

The final cost that must be considered is that of the surveillance programme itself. This is more simple to calculate and may be parametrised as constant s. Now that we have all the individual pieces, we can say that the optimal surveillance budget is that which minimises the following:

$$\underset{s \geq 0}{\text{minimise}} \, (s + \rho C(s)) \tag{A.10}$$

The benefit of considering the problem in terms of unique land parcels is that it allows us to easily account for the fact that different geographical locations will hold different likelihoods and consequences of an incursion and this information can be captured in the input parameters. An additional convenience of this model is that it is unit-free and the land parcels can be to any scale that we choose as long as the units are consistent across all the parameters.

B. Detection probability for scenario (b)

The proportion of the square that is covered by the circle is defined as

$$\text{Area proportion} = \frac{\text{Area of the circle} - \text{Area of the segments outside the square}}{\text{Area of the square}} \tag{B.1}$$

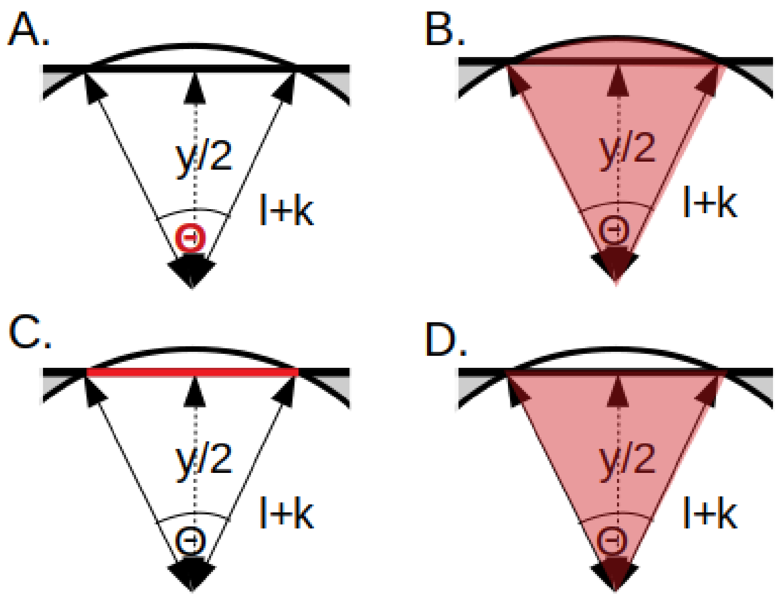


Figure B.1.: Steps for calculating the segment area.

To calculate the area of the segments outside the square, we first calculate the angle θ as displayed in Figure B.1 as

$$\theta = 2\cos^{-1}\left(\frac{y}{2(l+k)}\right) \tag{B.2}$$

Using this, we calculate the area of the sector as

$$\text{Sector area} = \frac{\theta(l+k)^2}{2} \tag{B.3}$$

We then calculate the length between the two points of intersection as

$$\text{Length} = \sqrt{(l+k)^2 - \left(\frac{y}{2}\right)^2} \tag{B.4}$$

Then we calculate the area of the triangle

$$\text{Triangle area} = \text{Length} \times \frac{y}{2} \tag{B.5}$$

Finally, we calculate the segment area as

$$\text{Segment area} = \text{Sector area} - \text{Triangle area} \tag{B.6}$$

We can multiply this by 4 to account for all sides of the grid cell so that the detection probability function for scenario (b) is

$$p(x,y) = \frac{\pi(l+k)^2 - 4\left(\frac{2\cos^{-1}\left(\frac{y}{2(l+k)}\right)(l+k)^2}{2} - \frac{y\sqrt{(l+k)^2-\left(\frac{y}{2}\right)^2}}{2}\right)}{y^2} \tag{B.7}$$

We simplify Equation B.8 to obtain

$$p(x,y) = \frac{\pi(l+k)^2 - 4\cos^{-1}\left(\frac{y}{2(l+k)}\right)(l+k)^2 + 2y\sqrt{(l+k)^2-\left(\frac{y}{2}\right)^2}}{y^2} \tag{B.8}$$